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New approach to thermal power plants operation regimes maximum power versus maximum efficiency

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Abstract

This work deals with the optimization of the thermal power plants based on Carnot exoirreversible cycle. The external irreversibility is due to the heat transfer at finite temperature differences between the working fluid and the external heat sources. Two fundamental operation regimes will be defined: (i) the power regime and (ii) the economical–ecological regime. The optimization of a thermal power plant with defined heat exchanger thermal conductances provides a "maximum power operation regime". At a given value of the total heat exchanger thermal conductance, imposed by the system size restriction, for different possible values of the total thermal conductance distribution, one results an infinite number of "maximum power regimes". Each of these regimes leads to a specific pair of optimal values for the temperature differences but keeps the same value for the thermal efficiency ($\eta_{opt} = 1 - \sqrt{T_0/T}$). Among the maximum power operation regimes, there is only one "maximum-maximorum power regime". In the maximum power regimes the entropy generation is not minimal. The "maximum-maximorum power regime" is characterized by the highest entropy generation. Another method of thermal power plant optimization with fuel consumption, respectively, input heat flux imposed, shows the existence of an "economical–ecological operation regime". This regime has a maximum thermal efficiency and a minimum entropy generation at corresponding maximum power production. The "economical–ecological" is obtained for a specific pair of optimal values for the temperature differences and demands the equipartition of the thermal conductances, corresponds to the optimal design condition. In the "economical–ecological regimes" characterized by low value of the temperature differences at the high temperature source, corresponding to supercritical thermodynamic conditions, the thermal power plant obtains a higher thermal efficiency than the optimal value of "power operating regimes". Among the infinite number of economical–ecological regimes, corresponding to different imposed values of the input heat flux, there is one with the highest power output—"the maximum-maximorum power regime".
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Keywords: Irreversible thermodynamics; Maximum power operation regime; Economical–ecological operation regime; Supercritical thermodynamic conditions

1. Introduction

The development of the irreversible thermodynamic analysis named "in finite time" is connected with the study of the conditions in which a thermal power plant (using a steam turbine and a classic or a nuclear vapor generator) can provide the maximum power. This method proposes an interdisciplinary approach (thermodynamics – heat transfer – fluids mechanics) that involves external irreversibility caused by the thermal and mechanical interactions at finite difference of potential between the system and the environment.

Thus, the analysis of the Carnot power cycle situated in the saturated humid vapors domain with thermal interactions at finite temperature differences between the agent and the external sources (Fig. 1), made by Penfield–Chambadal–Novikov–Curzon–Ahlborn shows that the pair of temperature differences values ΔT and ΔT_0 in the heat transfer processes with the two external sources (high with temperature T and low with temperature T_0), can be optimized for the power production maximization [1–5]. As a result, the operation of a thermal power plant with known design parameters (heat transfer surfaces for the production and condensation of vapors) at maximum power ($\dot{W}_{max}$) involves an optimal input heat flux ($\dot{Q}_{opt}$) from the high temperature source in association with the release toward the low temperature source of an optimal flux ($\dot{Q}_0^{opt}$). In these con-

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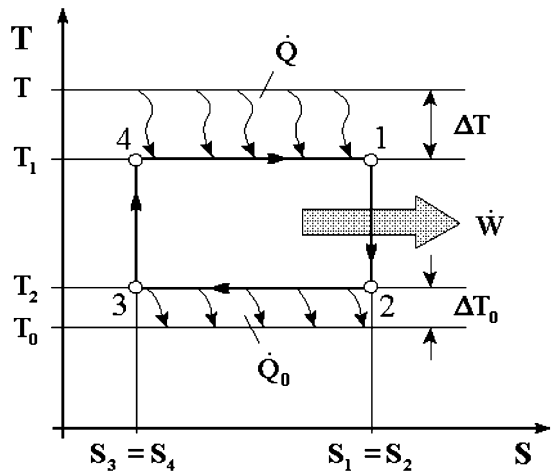

Fig. 1. Exoirreversible Carnot cycle in T–s diagram.

ditions, which may be named as "power operation regime", for an existing design configuration, the thermal efficiency is defined as a function of the "nice radical" ($\sqrt{T_0/T}$) [6,7] with the following relation:

$$\eta_{\text{opt}} = \frac{\dot{W}_{\max}}{\dot{Q}_{\text{opt}}} = 1 - \frac{\dot{Q}_0^{\text{opt}}}{\dot{Q}_{\text{opt}}} = \eta_{\text{PCANC}} = 1 - \sqrt{\frac{T_0}{T}} < \eta_C = 1 - \frac{T_0}{T} \tag{1.1}$$

in which η_C is the thermal efficiency of endo- and exoreversible Carnot power cycle, delimited by the same temperatures T and T_0, but with null temperature differences agent-external sources (ΔT, $\Delta T_0 \to 0$) and unlimited heat transfer surfaces (A, $A_0 \to \infty$). This means infinite time of contact between the agent and the heat sources. Therefore, the power production of the endo- and exoreversible Carnot cycle is zero.

Thus, based on finite time thermodynamics, even if these methods are sometimes subjected to criticisms [8,9], it was demonstrated for the first time that the thermal power plants can practically generate power only in the presence of the absolutely necessary external irreversibility. A thermodynamic cycle with external irreversibility may be named exoirreversible [7,10,11].

2. Optimization model of a thermal power plant

The aim of this thermodynamic analysis is to optimize the operation parameters (temperature differences between the working fluid and heat sources), design parameters (heat exchanger thermal conductances) and the exchanged heat fluxes of a thermal power plant, working on Carnot exoirreversible power cycle, in order to find (i) the power and (ii) the economical–ecological operation regime conditions. The T–s diagram of endoreversible and exoirreversible Carnot power cycle, situated in the saturated humid vapors domain between the temperatures $T_1 = T - \Delta T$ and $T_2 = T_0 + \Delta T$, is presented in Fig. 1.

With the heat transfer surfaces, respectively A of the vapors generator and A_0 of the condenser and also with the global heat transfer coefficients k and k_0, the thermal conductances of the system are given by the following relations:

$$K = kA; \qquad K_0 = k_0 A_0 \tag{2.1}$$

with K, K_0 the thermal conductances of the vapor generator and the condenser.

Thus, the thermal fluxes exchanged can be written as functions of conductances:

$$\dot{Q} = kA\Delta T = K\Delta T; \qquad \dot{Q}_0 = k_0 A_0 \Delta T_0 = K_0 \Delta T_0 \tag{2.2}$$

The development of the model is based on the energy and entropy flux balance equations at a constant value of the total heat exchanger thermal conductance ($K_t = ct$), imposed by the system size restriction. Using Eqs. (2.2) and the working fluid extreme temperatures, the initial equation system becomes:

$$\begin{cases} \dot{W} = \dot{Q} - \dot{Q}_0 \\ \dfrac{\dot{Q}}{T_1} = \dfrac{\dot{Q}_0}{T_2} \\ K + K_0 = K_t \end{cases} \Rightarrow \begin{cases} \dot{W} = K\Delta T - K_0 \Delta T_0 \\ \dfrac{K\Delta T}{T - \Delta T} = \dfrac{K_0 \Delta T_0}{T_0 + \Delta T_0} \\ K + K_0 = K_t \end{cases} \tag{2.3}$$

A generalized model can be obtained using the following dimensionless expressions:

$$\begin{aligned} &\overline{K} = \frac{K}{K_t}; \qquad K_0 = \frac{K_0}{K_t}; \qquad \tau = \frac{T}{T_0} \\ &\theta = \frac{\Delta T}{T}; \qquad \theta_0 = \frac{\Delta T_0}{T_0} \\ &\overline{W} = \frac{\dot{W}}{K_t T}; \qquad \overline{Q} = \frac{\dot{Q}}{K_t T} = \frac{K\Delta T}{K_t T} = \overline{K}\theta \\ &\overline{Q}_0 = \frac{\dot{Q}_0}{K_t T} = \frac{1}{\tau}\overline{K}_0\theta_0 \end{aligned} \tag{2.4}$$

Using the dimensionless parameters the equations system (2.3) becomes:

$$\begin{cases} \overline{W} = \overline{Q} - \overline{Q}_0 \\ \dfrac{\overline{Q}}{1-\theta} = \tau\dfrac{\overline{Q}_0}{1-\theta_0} \\ \overline{K} + \overline{K}_0 = 1 \end{cases} \Rightarrow \begin{cases} \overline{W} = \overline{K}\theta - \dfrac{1}{\tau}\overline{K}_0\theta_0 \\ \overline{K}\dfrac{\theta}{1-\theta} = \overline{K}_0\dfrac{\theta_0}{1-\theta_0} \\ \overline{K} + \overline{K}_0 = 1 \end{cases} \tag{2.5}$$

Considering an imposed value of the heat sources temperature ratio ($\tau = ct$), the system of Eqs. (2.5) will give the expression of the dimensionless power production as a function of four variables $\overline{W}(\overline{K}, \overline{K}_0, \theta, \theta_0)$ with two constraint equations: entropy balance and the total heat exchanger thermal conductance limitation.

The thermodynamic optimization study will be led in the following two steps:

(i) with $\tau = ct$, as imposed parameter, the maximization of the mechanical power $\overline{W}(\overline{K}, \overline{K}_0, \theta, \theta_0)$ at optimal thermal efficiency which is equivalent to "the power regime";

(ii) with $\tau = ct$ and imposed heat flux input $\overline{Q} = ct$, the maximization of the thermal efficiency $\eta = \frac{\overline{W}(\overline{K}, \overline{K}_0, \theta, \theta_0)}{\overline{Q}}$ which is equivalent to "the economical–ecological regime".

3. The power operation regime

From equations system (2.5), the power production function can be expressed by only two independent variables:

$$\overline{W}(\overline{K},\theta)=\overline{K}\theta\left(1-\frac{1}{\tau}\frac{1-\overline{K}}{1-\overline{K}-\theta}\right) \tag{3.1}$$

The exchanged thermal fluxes and the thermal efficiency can be similarly written:

$$\overline{Q}(K,\theta)=\frac{\dot{Q}}{K_tT}=\overline{K}\theta$$

$$\overline{Q}_0(K,\theta)=\frac{\dot{Q}_0}{K_tT}=\frac{1}{\tau}\frac{(1-\overline{K})\overline{K}\theta}{1-\overline{K}-\theta} \tag{3.2}$$

$$\eta(K,\theta)=\frac{\dot{W}}{\dot{Q}}=1-\frac{\dot{Q}_0}{\dot{Q}}=1-\frac{\overline{Q}_0}{\overline{Q}}=1-\frac{1}{\tau}\frac{1-\overline{K}}{1-\overline{K}-\theta} \tag{3.3}$$

Eq. (3.1) shows that the power production becomes null in two cases:

- if $\theta=\theta_1=0$; then $\overline{Q}=\overline{Q}_0=0$; $\overline{W}=0$; $\eta=\eta_C=1-1/\tau=1-T_0/T$;
- if $\theta=\theta_2=(1-\overline{K})(1-1/\tau)=(1-\overline{K})\eta_C$; then $\overline{Q}=\overline{Q}_0=(1-\overline{K})\overline{K}\eta_C$; $\overline{W}=0$; $\eta=0$.

It means that between the two extreme values of the parameter θ, there is an optimal value $\theta_{\mathrm{opt}}\in(\theta_1,\theta_2)$ for which the power becomes a maximum (the extremum $\mathrm{d}\overline{W}/\mathrm{d}\theta=0$ and the maximum $\mathrm{d}^2\overline{W}/\mathrm{d}\theta^2<0$ conditions for θ_{opt} are fulfilled). Thus, the extremum condition implies:

$$\theta_{\mathrm{opt}}=(1-\overline{K})\left(1-\sqrt{T_0/T}\right)=(1-\overline{K})\left(1-\sqrt{1/\tau}\right) \tag{3.4}$$

With this optimal value, one obtains the following expressions:

$$\eta_{\mathrm{opt}}=1-\sqrt{T_0/T}=1-1/\sqrt{\tau}<\eta_C=1-T_0/T=1-1/\tau \tag{3.5}$$

$$\begin{aligned}&\overline{W}_{\max}(\overline{K})=(1-\overline{K})\overline{K}\eta_{\mathrm{opt}}^2\\&\overline{Q}_{\mathrm{opt}}(\overline{K})=(1-\overline{K})\overline{K}\eta_{\mathrm{opt}};\qquad \overline{Q}_{0,\mathrm{opt}}(\overline{K})=\overline{Q}_{\mathrm{opt}}/\sqrt{\tau}\end{aligned} \tag{3.6}$$

The analysis of Eqs. (3.4)–(3.6) shows that the "maximal power regime" conditions are function of $\overline{K}$. For all values of $\overline{K}$ i.e. (0–1) in this study, there is an infinite number of "maximal power regimes", all of them with the same value of the thermal efficiency η_{opt}.

On the other hand, the function $\overline{W}_{\max}(\overline{K})$ has an optimal value resulting from the condition $\mathrm{d}\overline{W}_{\max}(\overline{K})/\mathrm{d}\overline{K}=0$, which gives the optimal values of the relative conductances:

$$(\overline{K})_{\mathrm{opt}}=(\overline{K}_0)_{\mathrm{opt}}=0.5 \tag{3.7}$$

Eq. (3.7) expresses the famous equipartition principle that corresponds to the optimal design condition [12,13].

In these conditions, one obtained in an original way the "maximum-maximorum power regime", very important in thermal power plant operation as a unique regime which is specified by the following conditions:

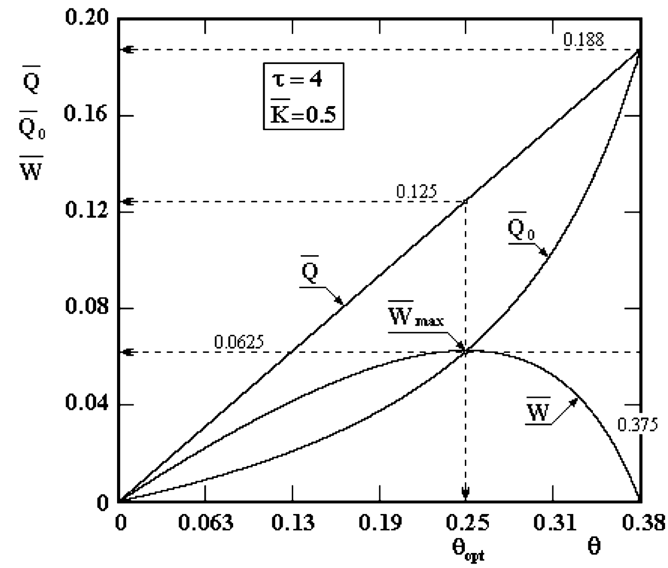

Fig. 2. Variation of $\overline{Q}$, $\overline{Q}_0$ and $\overline{W}$ as a function of θ; "maximum power regime" $\overline{W}_{\max}$ at θ_{opt}.

$$\begin{aligned}&\theta_{\mathrm{opt}}^{\mathrm{opt}}=0.5\eta_{\mathrm{opt}};\qquad \theta_{0,\mathrm{opt}}^{\mathrm{opt}}=\theta_{\mathrm{opt}}^{\mathrm{opt}}/\sqrt{\tau}\\&\overline{Q}_{\mathrm{opt}}^{\mathrm{opt}}=0.25\eta_{\mathrm{opt}};\qquad \overline{Q}_{0,\mathrm{opt}}^{\mathrm{opt}}=\overline{Q}_{\mathrm{opt}}^{\mathrm{opt}}/\sqrt{\tau}\\&\overline{W}_{\max}^{\max}=\overline{Q}_{\mathrm{opt}}^{\mathrm{opt}}-\overline{Q}_{0,\mathrm{opt}}^{\mathrm{opt}}=0.25\eta_{\mathrm{opt}}^2\\&T_{1,\mathrm{opt}}^{\mathrm{opt}}=T\left(1-\theta_{\mathrm{opt}}^{\mathrm{opt}}\right);\qquad T_{2,\mathrm{opt}}^{\mathrm{opt}}=T_0\left(1+\theta_{0,\mathrm{opt}}^{\mathrm{opt}}\right)\\&\eta_{\mathrm{opt}}=1-\sqrt{T_0/T}=1-T_{2,\mathrm{opt}}/T_{1,\mathrm{opt}}=1-T_{2,\mathrm{opt}}^{\mathrm{opt}}/T_{1,\mathrm{opt}}^{\mathrm{opt}}\end{aligned} \tag{3.8}$$

From Eqs. (3.8) it results that the increase of τ leads to the increase of the optimal value $\theta_{\mathrm{opt}}^{\mathrm{opt}}$ and also to higher performances, $\overline{W}_{\max}^{\max}$ and η_{opt}.

The diagram of Fig. 2 shows: $\overline{Q}=f(\theta)$, $\overline{Q}_0=f(\theta)$ and $\overline{W}=f(\theta)$ for $\tau=4$ and $\overline{K}=0.5$. In this diagram, "maximal power regime" appears with a maximal difference between $\overline{Q}$ and $\overline{Q}_0$ for $\theta=\theta_{\mathrm{opt}}$. Fig. 3 shows (for $\tau=4$) the variation of the power provided by the plant for three different values of $\overline{K}$ (0.2, 0.5 and 0.8), the "maximum-maximorum power regime" linked to $\overline{K}=\overline{K}_0=0.5$.

Fig. 3 also shows that maximal power increases with θ_{opt} in the domain $\overline{K}\in(0.2\text{–}0.5)$ and decreases in the domain $\overline{K}\in(0.5\text{–}0.8)$. It points out the symmetry (same $\overline{W}_{\max}=0.04$ for both $\overline{K}=0.2$ and $\overline{K}=0.8$) of the values locus of the maximal power $\overline{W}_{\max}$ in reference to the maximum-maximorum power value $\overline{W}_{\max}^{\max}=0.625$ (with $(\overline{K})_{\mathrm{opt}}=0.5$ and $\theta_{\mathrm{opt}}^{\mathrm{opt}}=0.25$).

Fig. 4 shows, for the previously specified values of $\overline{K}$ (0.2, 0.5 and 0.8), the variation of thermal efficiency $\eta=f(\theta)$ of the system (Eq. (3.3)). One point out the fact that all the families of "maximal power regimes" have the same efficiency $\eta_{\mathrm{opt}}=1-\sqrt{T_0/T}$. It is interesting to observe that all the curves cut the vertical axis at the same point $\theta=0$ and $\eta=\eta_C=1-T_0/T$ corresponding to $\overline{W}=0$.

At $\tau=ct$, for the "maximum-maximorum power regime" $\theta_{\mathrm{opt}}^{\mathrm{opt}}$ increases with the increase of $\overline{Q}$ (Fig. 5).

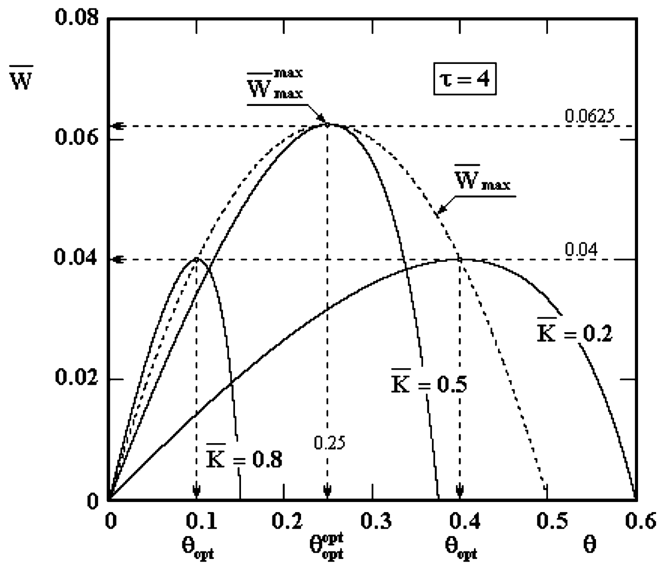

Fig. 3. Variation of $\overline{W}$ as a function of θ for: $\overline{K}=0.2$, $\overline{K}=0.5$ and $\overline{K}=0.8$; "maximum power regimes" $\overline{W}_{\max}$ at $\theta_{\rm opt}$; "maximum-maximorum regime" $\overline{W}^{\max}_{\max}$ at $(\overline{K})_{\rm opt}=0.5$ and $\theta^{\rm opt}_{\rm opt}$.

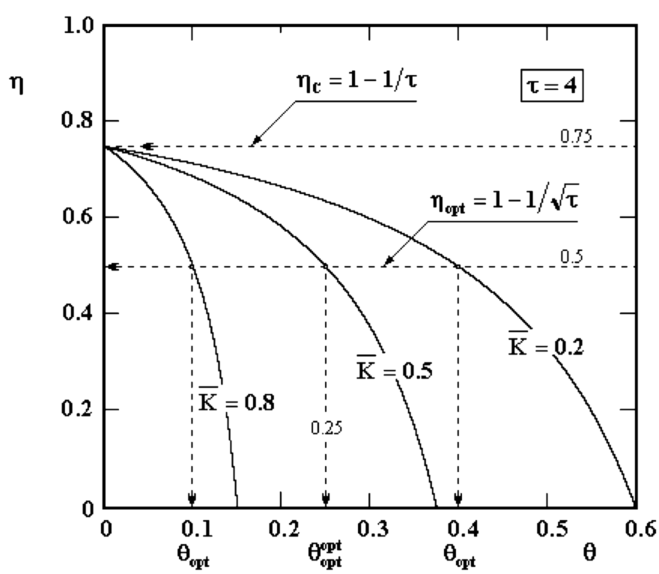

Fig. 4. Variation of η as a function of θ for: $\overline{K}=0.2, \overline{K}=0.5$ and $\overline{K}=0.8$; "maximum power regimes" $\eta_{\rm opt}$ at $\theta_{\rm opt}$; "maximum-maximorum power regime" at $(\overline{K})_{\rm opt}=0.5$ and $\theta^{\rm opt}_{\rm opt}$.

Thus, it is also interesting to analyze the entropy generation in the heat transfer processes at finite temperature differences ΔT and ΔT_0, at heat sources. This can be calculated as follows:

$$\sum \dot{S} = \dot{S}_{\Delta T} + \dot{S}_{\Delta T_0} = \left(\frac{1}{T_1} - \frac{1}{T}\right)\dot{Q} + \left(\frac{1}{T_0} - \frac{1}{T_2}\right)\dot{Q}_0 \quad (3.9)$$

Using Eqs. (2.4), Eq. (3.9) leads to the dimensionless generated entropy:

$$\sum \overline{S} = \frac{\sum S}{K_t} = \overline{K}\frac{\theta^2}{1-\theta} + \overline{K}_0\frac{\theta_0^2}{1+\theta_0} \quad (3.10)$$

The entropy generation plot for $\tau = 4$ and for the previous values of $\overline{K}$ (0.2, 0.5 and 0.8) is illustrated in Fig. 6. One notices

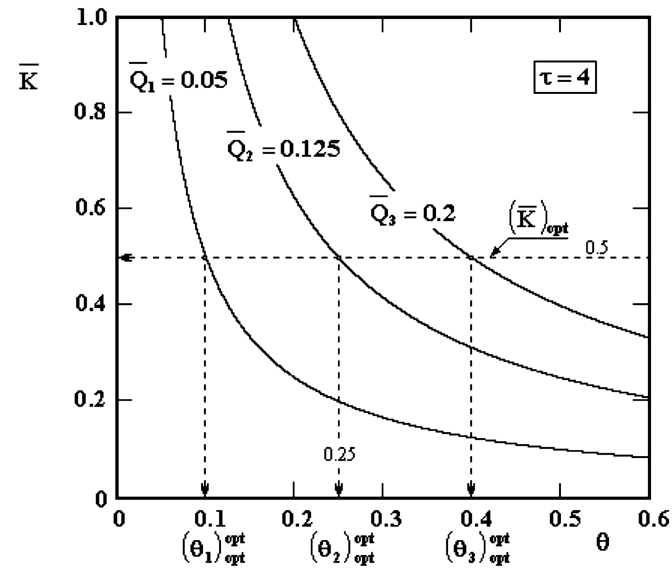

Fig. 5. Variation of $\theta^{\rm opt}_{\rm opt}$ as functions of $\overline{Q}$ for the "maximum-maximorum power regime" at $(\overline{K})_{\rm opt}=0.5$.

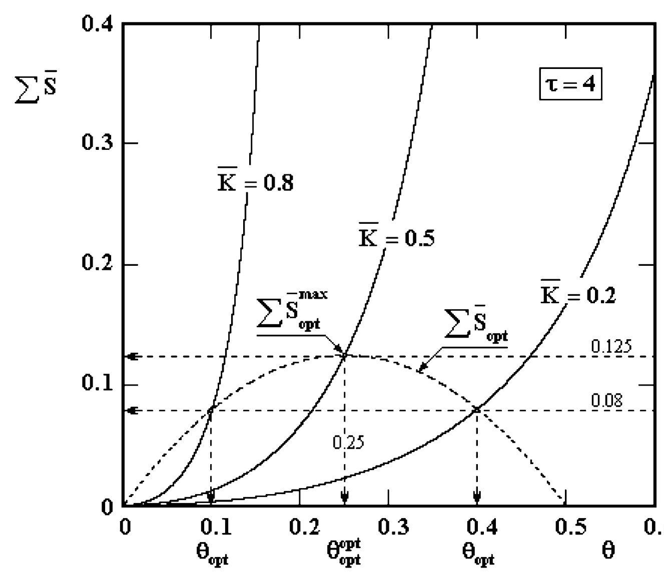

Fig. 6. Variation of $\sum\overline{S}$ as a function of θ for: $\overline{K}=0.2$, $\overline{K}=0.5$ and $\overline{K}=0.8$; "maximum power regimes" $\sum\overline{S}_{\rm opt}$ at $\theta_{\rm opt}$; "maximum-maximorum power regime" $\sum\overline{S}^{\max}_{\rm opt}$ at $(\overline{K})_{\rm opt}=0.5$ and $\theta^{\rm opt}_{\rm opt}$.

that for each of the three previous values of $\overline{K}$ there is a continuous increase of the entropy generation with θ. Moreover, it results that the entropy generation does not reach its minimum value when the thermal power plant produces maximal power. There is also a symmetry (equal values of $\sum\overline{S}_{\rm opt}=0.08$ at $\overline{K}=0.2$ and $\overline{K}=0.8$) of the entropy generation values locus $\sum\overline{S}_{\rm opt}$ in reference to the entropy generation in the maximum-maximorum power regime $\sum\overline{S}^{\max}_{\rm opt}=0.125$. This value is also a maximum, which means that "maximum-maximorum power regime" can be obtained only with a very strong entropy generation caused by the highest external irreversibility of the cycle.

The results of this first step of analysis emphasize a very important practical aspect of the thermal power plant: the maximal power production depends on the physical properties of the thermal agent. Having as example water with a critical temperature $t_{cr} = 374\,^\circ\mathrm{C}$ (647 K) and considering usual values for the high temperature source $T = 1200$ K (combustion gas) and for the low temperature source $T_0 = 300$ K (cooling water), meaning $\tau = 4$, the thermodynamic cycle occurring in the saturated humid vapor domain has a temperature difference $\Delta T \gg 300$ K, which means $\theta = \Delta T/T > 0.4$. In these conditions, to reach a maximal power regime $\overline{W}_{\max}$, a low value of $\overline{K}$ must be chosen, for example $\overline{K} < 0.2$ (see Fig. 3). Thus, to obtain the maximum-maximorum power regime corresponding to lower values of $\theta_{\mathrm{opt}}^{\mathrm{opt}}$, with reduced values for the dimensionless heat flux $\overline{Q}$ received from the high temperature source (Fig. 5), there is a need of a lower ΔT. This means that the vapor generator should work in a supercritical regime.

4. The economical–ecological operation regime

The aim of this second step of thermodynamic optimization study is to find the "economical operation regime" conditions in which the thermal power plant based on exoirreversible Carnot cycle (Fig. 1) has the maximum thermal efficiency for an imposed heat flux $\dot{Q}$ received from the high temperature source (for example, from the combustion gas obtained in the burning process of a fuel with a known lower calorific value). With this supplementary constraint, the equations system (2.5) can be written as follows:

$$\begin{cases} \eta = \dfrac{\overline{W}}{\overline{Q}} = 1 - \dfrac{\overline{Q}_0}{\overline{Q}} \\ \dfrac{\overline{Q}}{1-\theta} = \tau \dfrac{\overline{Q}_0}{1-\theta_0} \\ \overline{Q} = \overline{K}\theta \\ \overline{K} + \overline{K}_0 = 1 \end{cases} \quad \Rightarrow \quad \begin{cases} \eta = 1 - \dfrac{1}{\tau}\dfrac{\overline{K}_0\theta_0}{\overline{Q}} \\ \overline{K}\dfrac{\theta}{1-\theta} = \overline{K}_0\dfrac{\theta_0}{1-\theta_0} \\ \overline{Q} = \overline{K}\theta \\ \overline{K} + \overline{K}_0 = 1 \end{cases} \tag{4.1}$$

From Eqs. (4.1) the thermal efficiency η and the dimensionless heat flux evacuated $\overline{Q}_0$ can be written as functions of one variable as follows:

$$\eta(\theta) = 1 - \frac{1}{\tau}\frac{1}{1 - \theta^2/(\theta - \overline{Q})} \tag{4.2}$$

$$\overline{Q}_0(\theta) = \frac{1}{\tau}\frac{\overline{Q}}{1 - \theta^2/(\theta - \overline{Q})} \tag{4.3}$$

Changing the θ variable with a new one:

$$\varphi = \theta^2/\bigl(\theta - \overline{Q}\bigr) \tag{4.4}$$

Eqs. (4.1)–(4.3) and the corresponding power production become, respectively:

$$\begin{aligned} &\eta(\varphi) = 1 - \frac{1}{\tau(1-\varphi)}; \qquad \overline{Q}_0(\varphi) = \frac{\overline{Q}}{\tau(1-\varphi)} \\ &\overline{W}(\varphi) = \overline{Q}\left(1 - \frac{1}{\tau(1-\varphi)}\right); \qquad \theta_0(\varphi) = \frac{1}{\theta}\frac{\overline{Q}}{\varphi^{-1} - 1} \end{aligned} \tag{4.5}$$

The condition of physical compatibility with the problem proposed is $\tau(1-\varphi) > 1$, or $\varphi < (1 - 1/\tau = \eta_C)$.

Eqs. (4.5) show that an "economical regime" ($\eta_{\max}$ and corresponding $\overline{Q}_0^{\min}$, $\overline{W}_{\max}$) for a given value of $\overline{Q}$ parameter can be obtained for a minimal value of φ, i.e. for an optimal value of θ.

Solving the equation $\mathrm{d}\varphi/\mathrm{d}\theta = 0$, and verifying that $\mathrm{d}^2\varphi/\mathrm{d}\theta^2 > 0$, one obtains:

$$\varphi_{\min} = 4\overline{Q} \quad \text{and} \quad \theta_{\mathrm{opt}} = 2\overline{Q} \tag{4.6}$$

In these conditions, as $\overline{Q} = (\overline{K})_{\mathrm{opt}}\theta_{\mathrm{opt}}$, according to the system size restriction, the "economical regime" (same as the "maximum-maximorum regime") demands the equipartition of the thermal conductances:

$$(\overline{K})_{\mathrm{opt}} = (\overline{K}_0)_{\mathrm{opt}} = 0.5 \tag{4.7}$$

And thus for the "economical operation regime" one obtains:

$$\begin{aligned} &\eta_{\max} = 1 - \frac{1}{\tau}\frac{1}{1-4\overline{Q}}; \qquad \overline{Q}_0^{\min} = \frac{1}{\tau}\frac{\overline{Q}}{1-4\overline{Q}} \\ &\overline{W}_{\max} = \overline{Q}\left(1 - \frac{1}{\tau}\frac{1}{1-4\overline{Q}}\right) \\ &\theta_0^{\mathrm{opt}} = \frac{2\overline{Q}}{1-4\overline{Q}} = \frac{\theta_{\mathrm{opt}}}{1-4\overline{Q}} \end{aligned} \tag{4.8}$$

From Eq. (4.4), at $\varphi > 0$, it results the condition $\theta > \overline{Q}$. Also, from Eqs. (4.8), for $\overline{Q} < 0.25$, it results $\theta_0^{\mathrm{opt}} > \theta_{\mathrm{opt}}$, that implies $\Delta T_0^{\mathrm{opt}} > \Delta T_{\mathrm{opt}}/\tau$, corresponding to an supercritical thermodynamic cycle.

Estimation of the reversibility degree can be done using the exergy efficiency defined by the equation:

$$\eta_{\mathrm{ex}}(\varphi) = \frac{\eta}{\eta_C} = \frac{1 - \frac{1}{\tau(1-\varphi)}}{1 - 1/\tau} \tag{4.9}$$

According to (4.9), with $\varphi \equiv \varphi_{\min} = 4\overline{Q}$, for $\tau = ct$, the maximal value of the exergy efficiency is:

$$\eta_{\mathrm{ex}}^{\max} = \frac{1 - \frac{1}{\tau(1-4\overline{Q})}}{1 - 1/\tau} \tag{4.10}$$

The maximum value of the exergetic efficiency always corresponds to minimum entropy generation [14]. Based on this remark, similarly to the "power regime" analysis, it is important to study the entropy generation in the thermal interactions at finite temperature differences ΔT and ΔT_0 at heat sources, also for the "economical regime". We must adapt Eq. (3.10) to this case; using the new imposed dimensionless parameter $\overline{Q}$ and Eqs. (4.5) the dimensionless entropy generation will be:

$$\sum \overline{S}(\theta) = \overline{Q}\frac{\theta}{1-\theta} + \left(1 - \frac{\overline{Q}}{\theta}\right)\left(\frac{\overline{Q}/\theta}{\varphi^{-1}-1}\right)^2 \Bigg/ \left(1 + \frac{\overline{Q}/\theta}{\varphi^{-1}-1}\right) \tag{4.11}$$

The "ecological regime" with minimal entropy generation can be found by solving the equation $\mathrm{d}\sum\overline{S}/\mathrm{d}\theta = 0$, while $\mathrm{d}^2\sum\overline{S}/\mathrm{d}\theta^2 > 0$, which leads to the optimal values $\theta \equiv \theta_{\mathrm{opt}} = 2\overline{Q}$. The conclusion is that the "economical regime" with maximal efficiency $\eta_{\max}$ is also an "ecological regime" with minimum entropy generation, that is why this regime is proposed to be named as "economical–ecological operation regime".

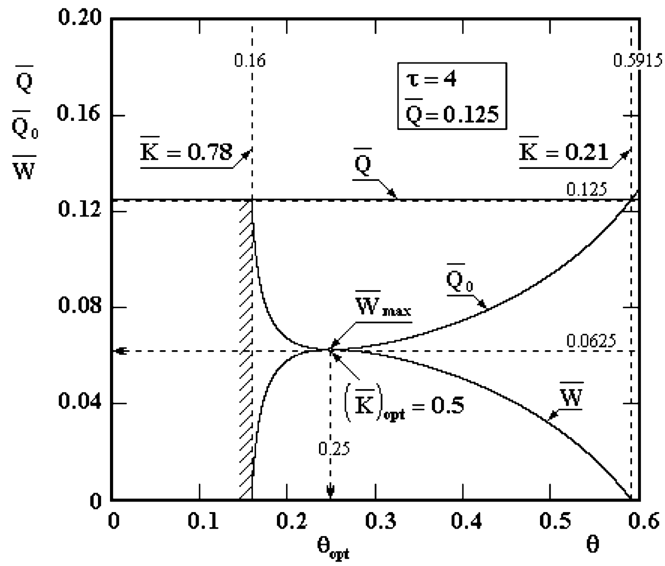

Fig. 7. Variation of $\overline{Q}$, $\overline{Q}_0$ and $\overline{W}$ as a function of θ; "economical–ecological regime" $\overline{W}_{\max}$ at θ_{opt}.

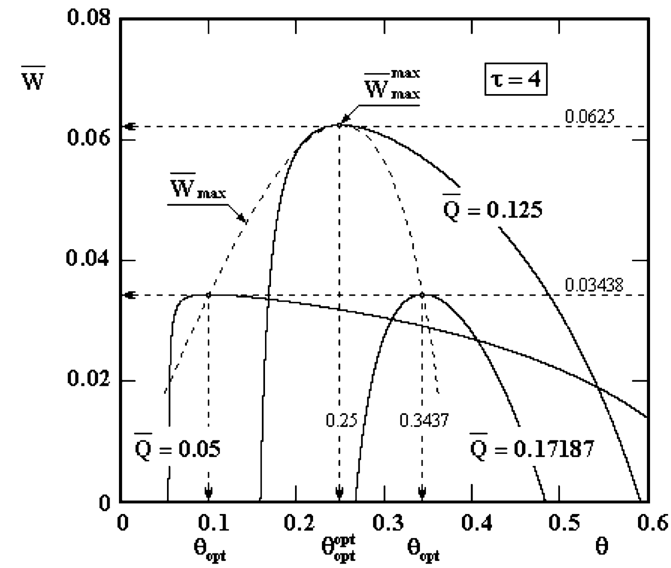

Fig. 8. Variation of $\overline{W}$ as a function of θ for: $\overline{Q}=0.05$, $\overline{Q}=0.125$ and $\overline{Q}=0.17187$; "economical regimes" $\overline{W}_{\max}$ at θ_{opt}; "maximum-maximorum regime" $\overline{W}_{\max}^{\max}$ at $(\overline{K})_{\text{opt}}=0.5$ and $\theta_{\text{opt}}^{\text{opt}}$.

In this case, at $\varphi\equiv\varphi_{\min}=4\overline{Q}$, the minimal entropy generation can be written using Eq. (4.11):

$$\sum\overline{S}_{\min}=\frac{4\overline{Q}^2}{1-4\overline{Q}} \tag{4.12}$$

Similarly to the case of the previous optimization study, next one does a parametric sensibility analysis for the second step optimization study.

To obtain a compatible value for $\overline{Q}$, which can be imposed in the second analysis, for example, a condition of equality between the thermal efficiency of an "economical–ecological regime" (Eq. (4.8)) and the thermal efficiency of a "maximum power regime" (Eq. (3.5)) can be applied:

$$\eta_{\max}=\eta_{\text{opt}}\quad\Rightarrow\quad\overline{Q}=\left(1-1/\sqrt{\tau}\right)/4 \tag{4.13}$$

and for $\tau=4$, one obtains $\overline{Q}=0.125$.

With these values, $\overline{Q}$, $\overline{Q}_0$ and $\overline{W}$ are represented as functions of the parameter θ (Fig. 7). The "economical–ecological regime" for the imposed value $\overline{Q}$ always corresponds to $(\overline{K})_{\text{opt}}=0.5$ (see Fig. 5) with $\theta_{\text{opt}}=0.25$. This regime is characterized by the maximal power production $\overline{W}_{\max}=0.0625$.

The Fig. 8, for $\tau=4$, shows the variation of the power production $\overline{W}$ for imposed values of $\overline{Q}$ (0.05, 0.125 and 0.17187). It results that for each imposed value of $\overline{Q}$ there is an infinite number of "economical–ecological regimes". According to Eq. (4.8), these different regimes correspond to different values of the maximal power ($\overline{W}_{\max}$) and respectively to different optimal values of θ_{opt}. We also point out the economical–ecological regime with the highest power ($\overline{W}_{\max}^{\max}$) which was called in the first step of the optimization study as "maximum-maximorum power regime". For $\overline{Q}\in(0.01\text{–}0.125)$, the power production increases and decreases for $\overline{Q}\in(0.125\text{–}0.19)$, as θ_{opt} increases. One also notices the symmetry (equal values of $\overline{W}_{\max}=0.03438$ at $\overline{Q}=0.05$ and $\overline{Q}=0.17187$) of the maximal power production ($\overline{W}_{\max}$) values locus in reference to maximum-maximorum power value ($\overline{W}_{\max}^{\max}=0.625$) at $\theta_{\text{opt}}^{\text{opt}}=0.25$.

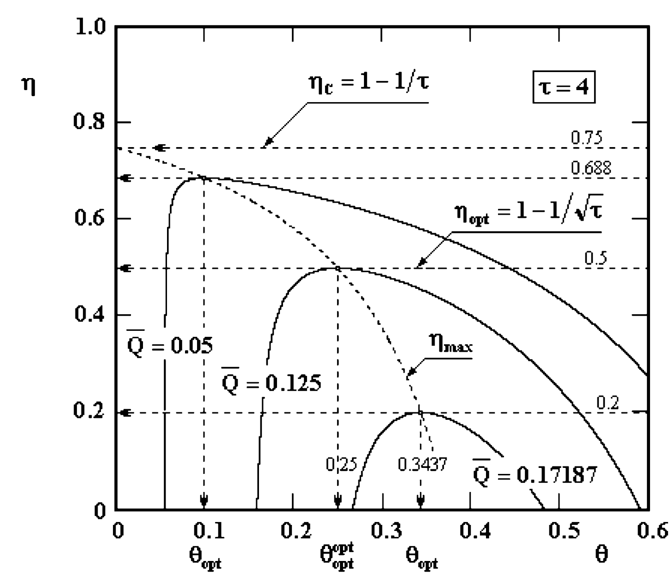

Fig. 9. Variation of η as a function of θ for: $\overline{Q}=0.05$, $\overline{Q}=0.125$ and $\overline{Q}=0.17187$; "economical regimes" $\eta_{\max}$ at θ_{opt}.

Fig. 9 shows, for $\tau=4$, the variation of the thermal efficiency for the previous values of $\overline{Q}$ (0.05, 0.125 and 0.17187) as a function of θ. For each imposed value of $\overline{Q}$, one obtains an "economical–ecological regime" corresponding to θ_{opt}. The locus of $\eta_{\max}$ increases towards the Carnot cycle thermal efficiency limit ($\eta_C=0.75$), for which the imposed received heat flux tends to zero. Also, we must point out that the thermal efficiency of the "maximum-maximorum regime" ($\eta_{\text{opt}}=0.5$) is not a maximum. Thus, for imposed values of $\overline{Q}\in(0\text{–}0.125)$, all the "economical–ecological regimes" have $\eta_{\max}>\eta_{\text{opt}}$. However, their power production is lower ($\overline{W}_{\max}<\overline{W}_{\max}^{\max}$).

According to Eq. (4.11), at $\tau=4$ and for the previous values of $\overline{Q}$ (0.05, 0.125 and 0.17187), the entropy generation in the "economical–ecological regime" is represented in the

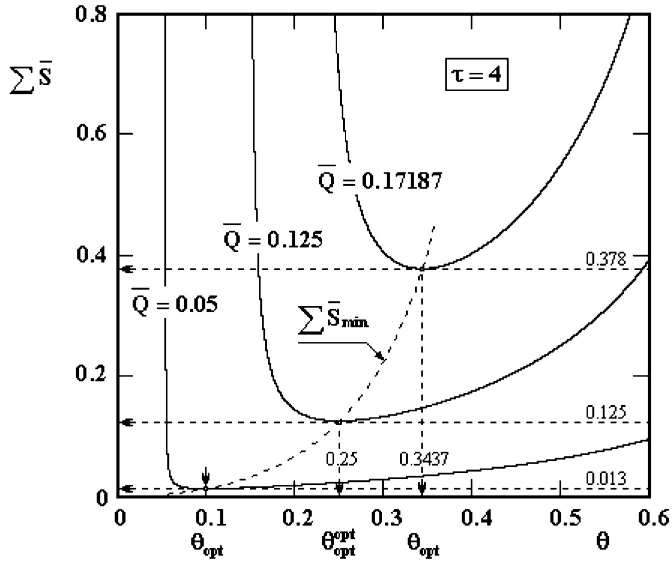

Fig. 10. Variation of $\sum\bar{S}$ as a function of θ for: $\bar{Q} = 0.05$, $\bar{Q} = 0.125$ and $\bar{Q} = 0.17187$; "economical–ecological regimes" $\sum\bar{S}_{\min}$ at θ_{opt}.

Fig. 10. One sees that for each value of imposed $\bar{Q}$, the economical regime with θ_{opt} has a minimal entropy generation $\sum\bar{S}_{\min}$. The locus of minimal entropy generation ($\sum\bar{S}_{\min}$) is increasing continuously while the input heat flux increases. It is very important to note that the minimal entropy generation locus ($\sum\bar{S}_{\min}$) is growing faster if the input heat flux exceeds the value that corresponds to "maximum-maximorum power regime" ($\bar{Q} > 0.125$).

As a conclusion summary of Figs. 9 and 10, comparing different "economical–ecological regimes", one observes a decrease of their thermal efficiency and an increase of the entropy generation if the value of the imposed parameter $\bar{Q}$ increases.

5. Conclusions

This paper covers different issues with direct implications in the field of thermal power plant optimization.

Based on the optimization model of a Carnot exoirreversible power cycle situated in the saturated humid vapors domain, with source temperature ratio ($\tau = T/T_0 = ct$) and total heat exchanger thermal conductance ($K_t = K + K_0 = ct$) as imposed parameters, there were defined two fundamental thermal power plant operation regimes: (i) the "power regime" and (ii) the "economical–ecological regime".

Thus, in the first case, the analysis of the thermal power plant operation reveals the "maximum power operation regimes" ($\bar{W}_{\max}$) depending on the optimal finite temperature differences as well as on different distribution of the thermal conductances. All these regimes have the same optimal thermal efficiency ($\eta_{\text{opt}} = 1 - \sqrt{T_0/T}$). From these "maximal power regimes", the one with the highest power production is called "maximum-maximorum power operation regime" ($\bar{W}_{\max}^{\max}$) and needs an equipartition of total heat exchanger thermal conductance $(\bar{K})_{\text{opt}} = (\bar{K}_0)_{\text{opt}} = 0.5$. All the "maximal power regimes" are not associated to minimum entropy generation. There is an optimal entropy generation for each regime ($\sum\bar{S}_{\text{opt}}$). Moreover, the "maximum-maximorum power regime" has the largest entropy generation ($\sum\bar{S}_{\text{opt}}^{\max}$) of all.

In the second case, the optimization study of the exoirreversible thermal power plant, with the imposed heat flux at the vapors generator ($\bar{Q}$) as given parameter, points out a unique "economical operation regime" for each value of $\bar{Q}$. In the "economical regimes", the thermal and exergy efficiencies are maximum ($\eta_{\max}$, $\eta_{\text{ex}}^{\max}$), related also to maximum power production ($\bar{W}_{\max}$). All "economical regimes" are characterized by the equipartition of the total heat exchanger thermal conductance, the optimal design condition. Also, the "economical regimes" have a minimum-ecological entropy generation ($\sum\bar{S}_{\min}$), therefore is proposed to be called the "economical–ecological operation regimes".

As consequence of the imposed value for $\bar{Q}$, taken as example in the second step optimization study, between the "power regimes" and the "economical–ecological regimes" there is only one common operation regime called "maximum-maximorum power regime" which has the highest power production, an optimal thermal efficiency ($\eta_{\text{opt}} = \eta_{\text{PCANC}}$) but the largest entropy generation of all "power regimes".

Real operating conditions of the thermal power plants, considering the thermo physical properties of water vapors, point out that both the "maximum-maximorum power regime" and the "economical–ecological regimes" impose a supercritical operation regime. In the past, in order to avoid the high pressure and temperature values of this regime, there was accepted the compromise of unfavorable working conditions (that implies high energy losses) over high temperature difference between the water vapors and the combustion gases of the vapor generator.

In present days, using the highest technologies, in order to reach the maximum-maximorum power and economical–ecological operation regimes, more and more thermal power plants are designed to operate in supercritical and advanced supercritical thermodynamic conditions [15,16].

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References

[1] P. Penfield Jr., Available power from a nonideal thermal source, J. Appl. Phys. 32 (9) (1961) 1793–1794.
[2] L. Curzon, B. Ahlborn, Efficiency of a Carnot engine at maximum power output, Am. J. Phys. 43 (1975) 22–24.
[3] I. Novikov, The efficiency of atomic power stations, translated from, Atomnaya Energiya 11 (1958) 409.
[4] P. Chambadal, Les centrales nucléaires, Armand Colin, Paris, 1957, pp. 41–58.
[5] M. El-Wakil, Nuclear Power Engineering, McGraw-Hill Book Company, New York, 1962, pp. 162–165.

[6] M. Feidt, Thermodynamique et optimisation énergétique des systèmes et procédés, Technique et Documentation, Lavoisier, Paris, 1987.

[7] A. Bejan, Advanced Engineering Thermodynamics, John Wiley & Sons, New York, 2006.

[8] M.J. Moran, in: A. Bejan, M. Feidt, M.J. Moran, G. Tsatsaronis (Eds.), Proceedings of ECOS'98, Nancy, France, 1998, pp. 1147–1150.

[9] V.M. Brodianskii, One step forward, tow step backward. The Second Law Analysis and the concept of Entropy Generation Minimization, in: Papers of the Int. Seminar: Contemporary Problems of Thermal Engineering, Gliwice – Ustron, Poland, 2–4 September 1998, pp. 37–46.

[10] V. Radcenco, Generalized Thermodynamics, Technique Edition, Bucharest, 1994.

[11] V. Radcenco, A Thermodynamics Theory of Physiques Interactions, Romanian Academy Edition, Bucharest, 2001 (in Romanian).

[12] A. Bejan, D. Tondeur, Equipartition, optimal allocation, and the constructal approach to predicting organization in nature, Rev. Gen. Thermique 37 (1998) 165–180.

[13] A. Bejan, Shape and Structure, from Engineering to Nature, Cambridge Univ. Press, Cambridge, UK, 2000.

[14] A. Bejan, Entropy Generation Minimization, CRC Press, Boca Raton, FL, 1996.

[15] K.M. Retzlaff, G. Schlottner, in: Steam Turbine for Ultrasupercritical Power Plants, POWER-GEN '98 Conference, Milan, June 9–11, 1998.

[16] I. Oprea, Steam and Gas Turbines, Printech Press, Bucharest, 2004 (in Romanian).